VISUAL 8.1 ▲ Warm-Up

1. Using your graphing calculator, graph the function

$$f(x) = 33x^3 - 250x^2 + 700x + 300.$$

Graph this function in the first quadrant viewing rectangle of [0, 10] [0, 3000].

2. In the same window find and graph the first derivative. Algebra II students will get this function from their teacher.

3. Explain how the second function relates to the first function.
(Algebra II students: Find a relationship by visually inspecting the graphs.)

4. Calculate the second derivative of f(x) and find the x value that makes it zero.
(Algebra II students: Get this function from your teacher.)

5. What is the meaning of the x value found in question 4 with respect to f(x) and f′(x)?

6. Sketch f(x) and f′(x) on the coordinate axes below. Use only the first quadrant.
Let each unit on the vertical axis represent 300.

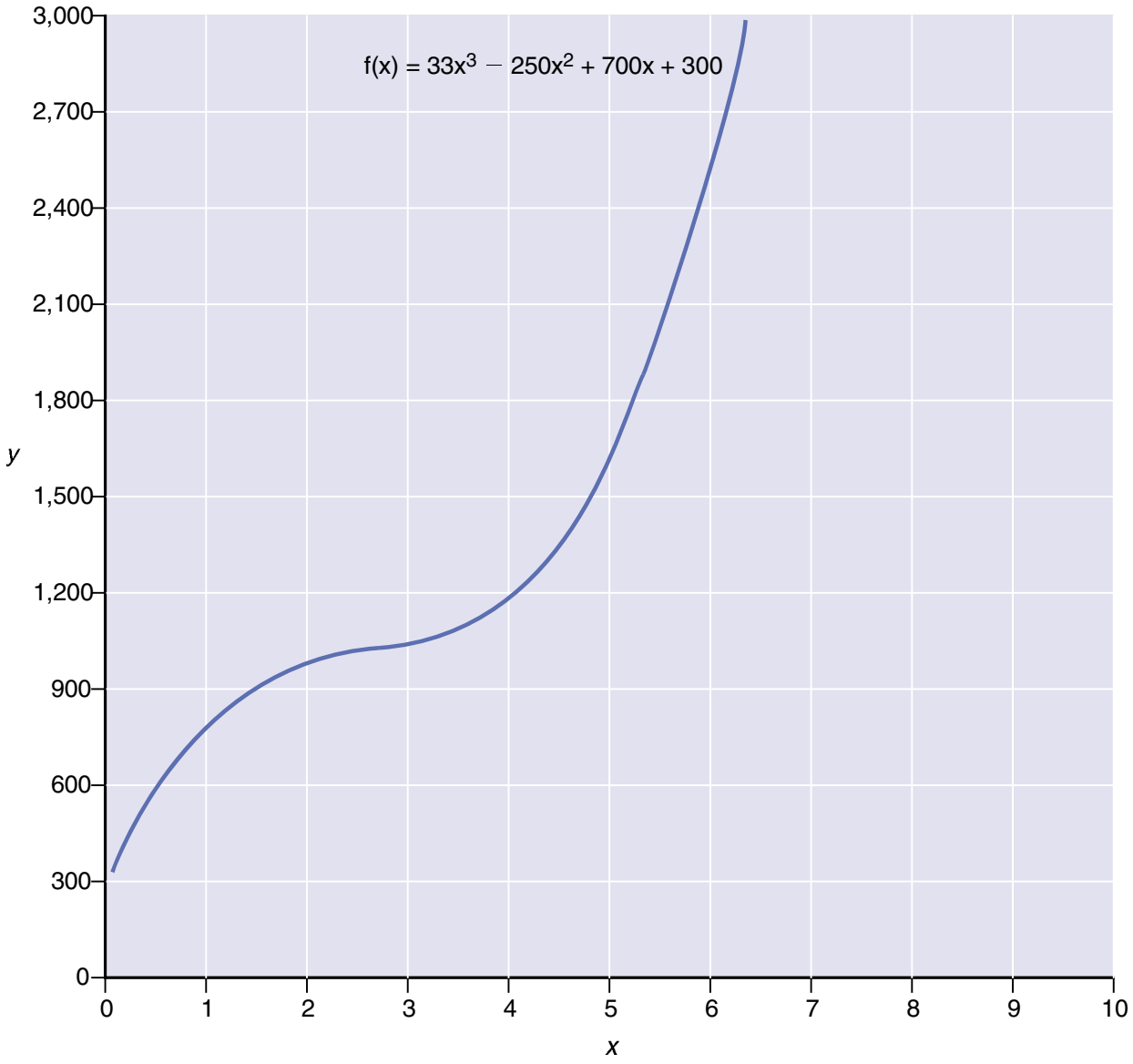
Graph of Function f(x) and its 1st Derivative

VISUAL 8.1 (continued)

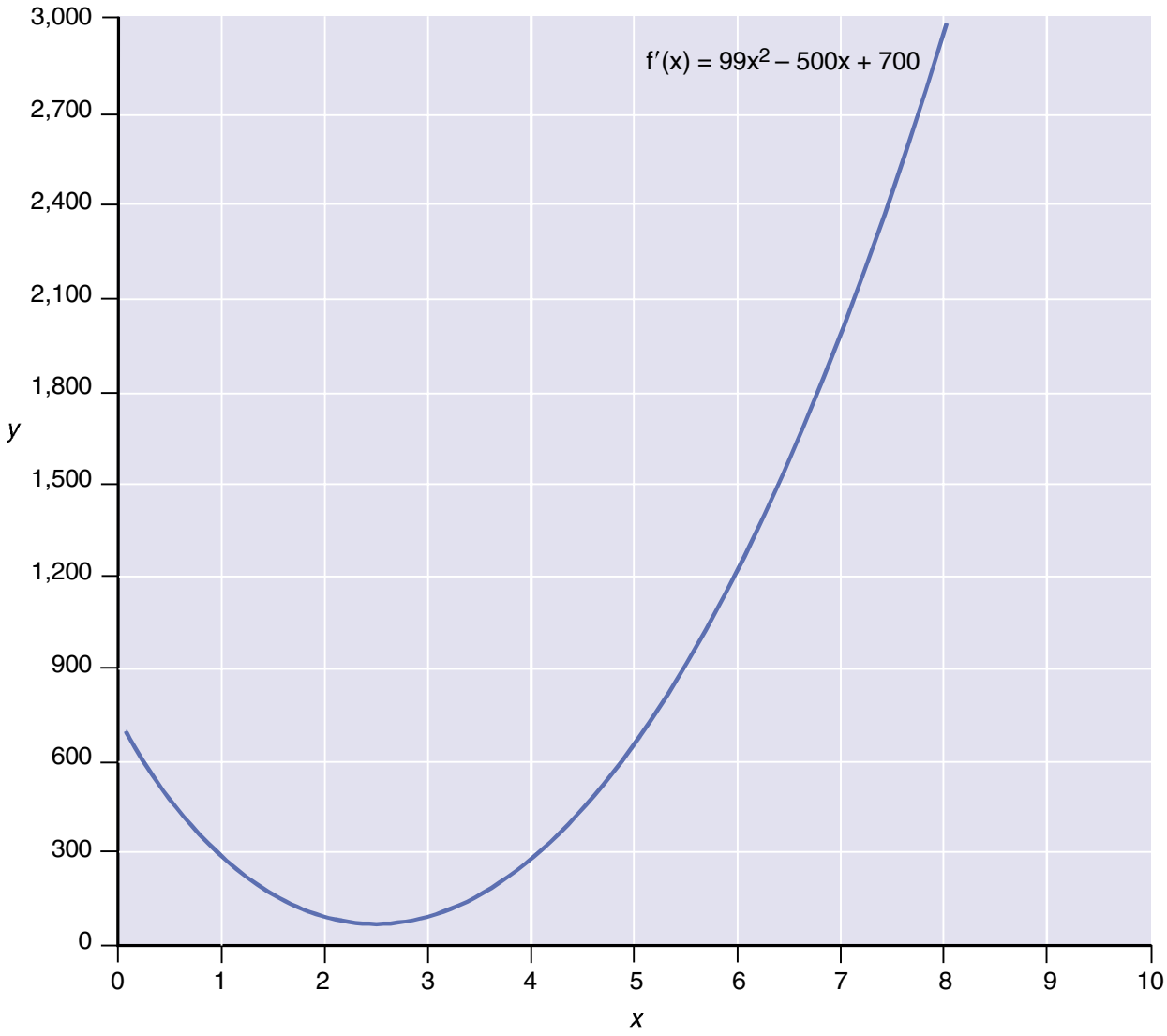
7. Calculate the slope of the curve f(x) by finding f′(0), f′(1), f′(2.5252. . .) and f′(3).

8. What is the meaning of the negative slope on the graph of f″(x) for $0 < x < 2.52$? (Algebra II students: Recall that f′(x) is a function that describes the slope of f(x). Examine your graphs.)

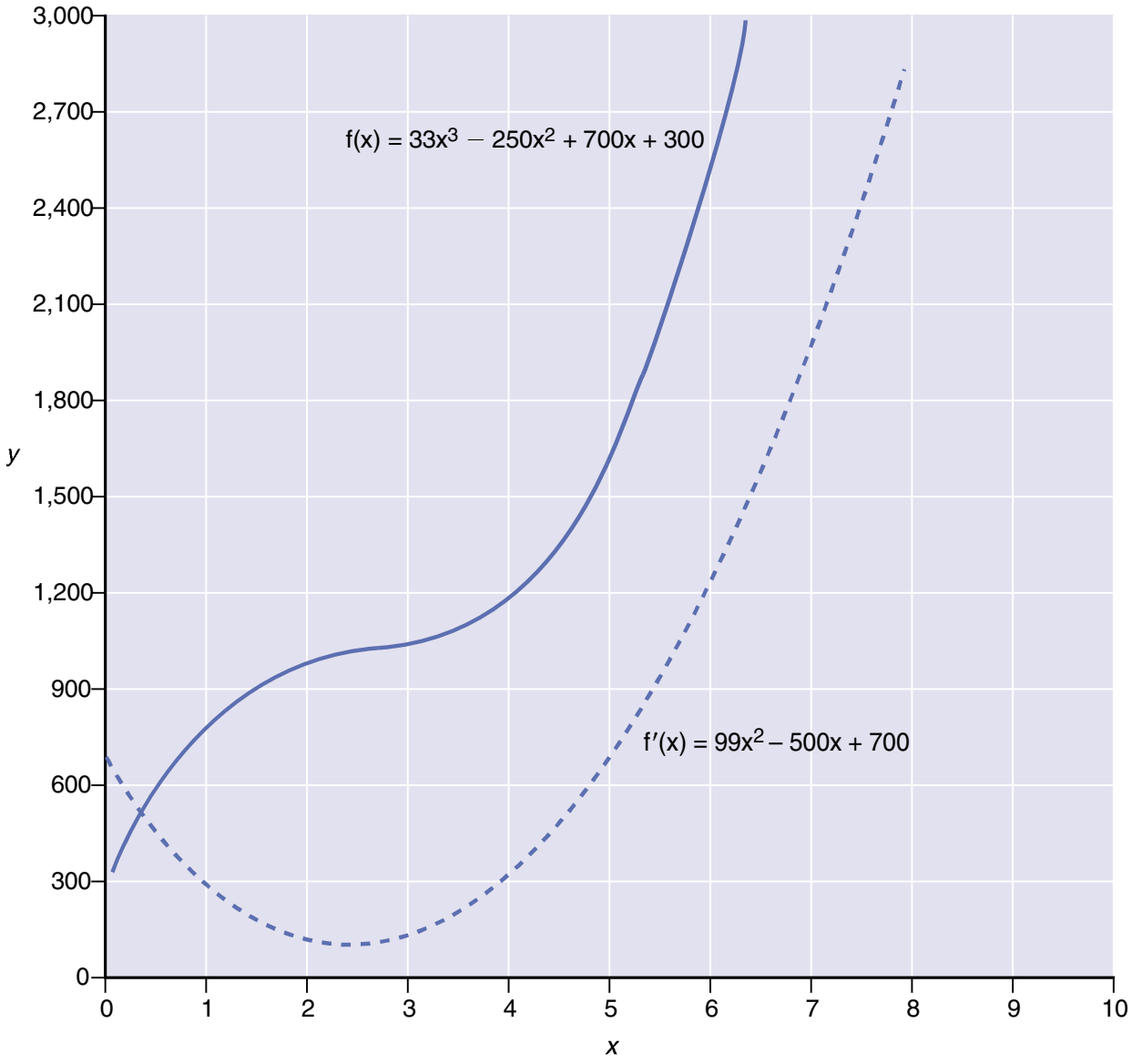
VISUAL 8.2


f(x) = 33x3 − 250x2 + 700x + 300
3,000
2,700
2,400
2,100
1,800
1,500
1,200
900
600
300
0
y
0
1
2
3
4
5
6
7
8
9
10
x

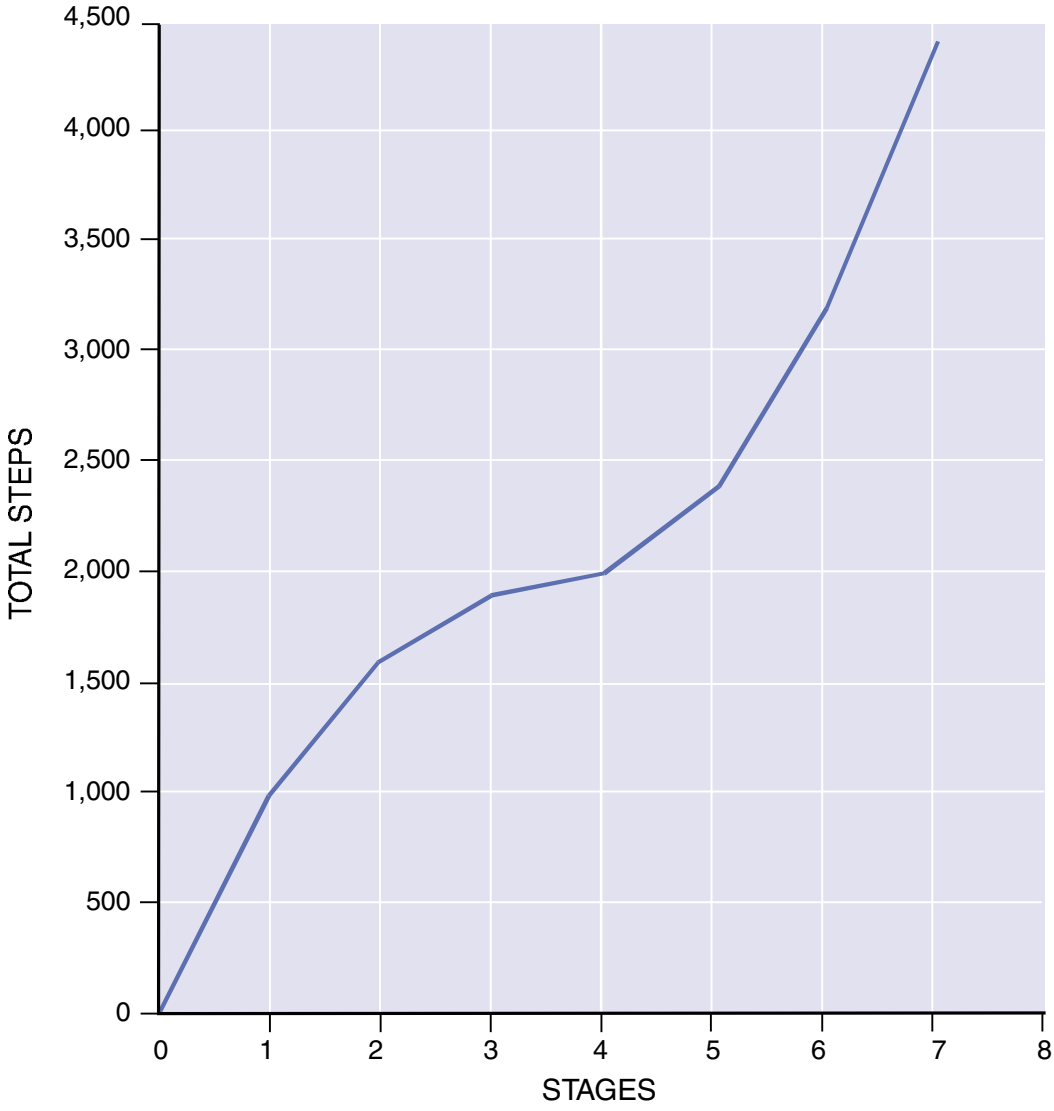
VISUAL 8.3



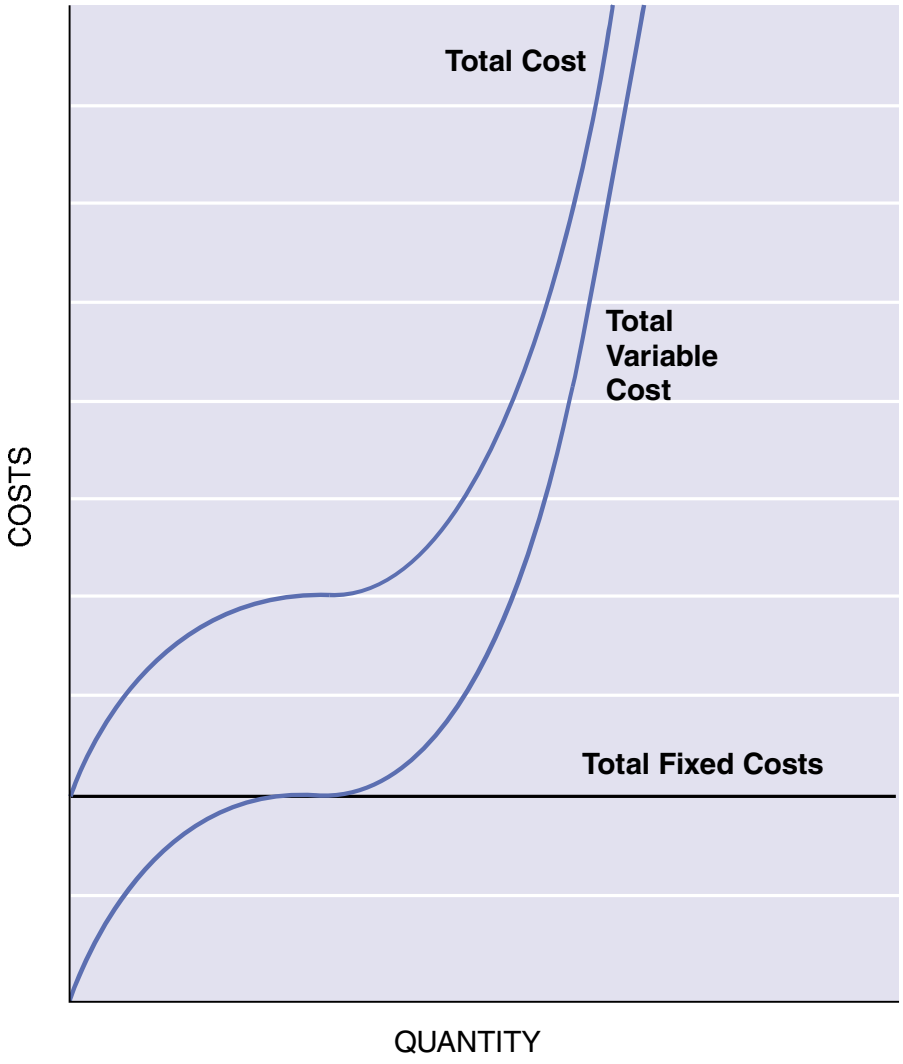
VISUAL 8.4



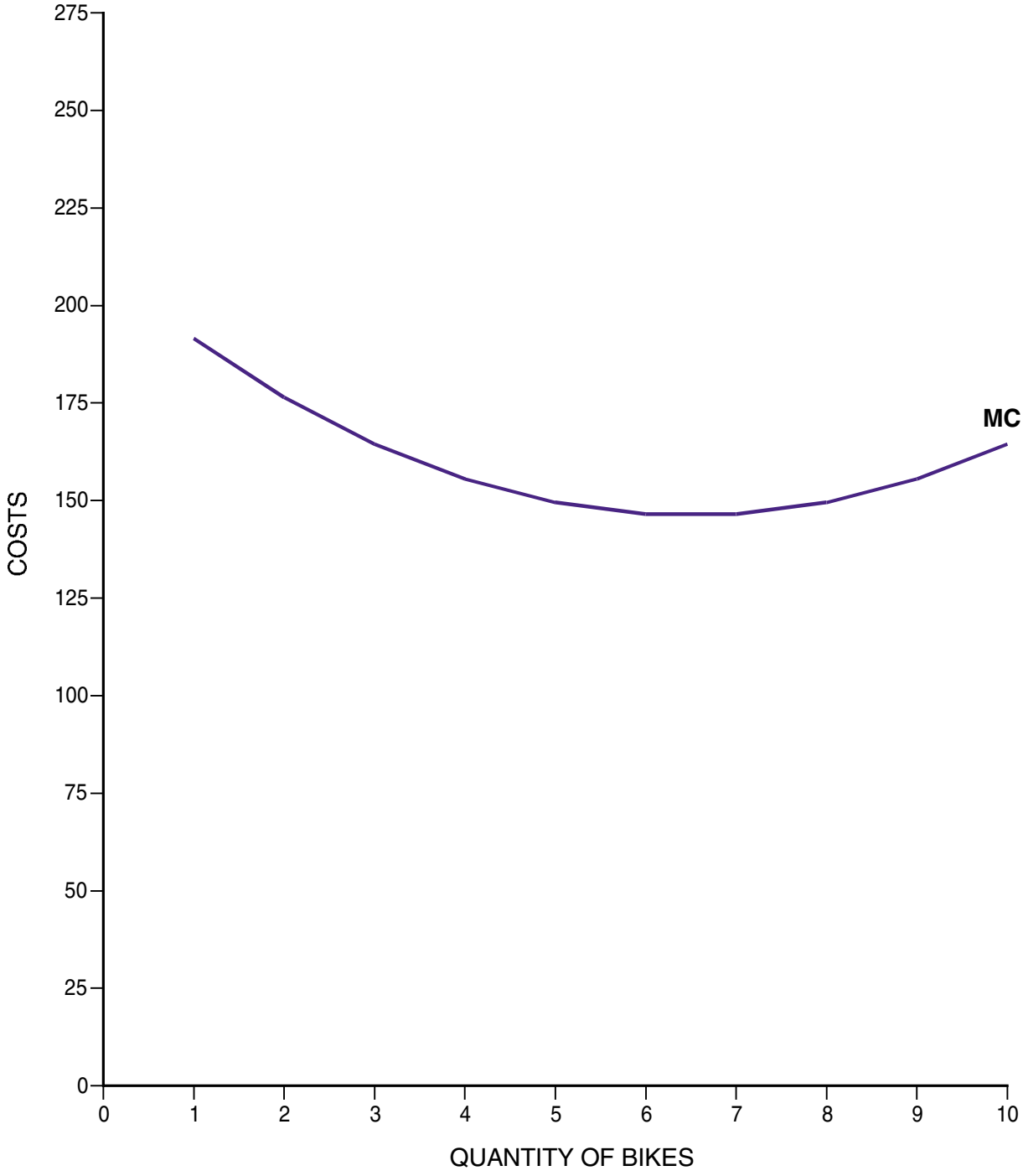
VISUAL 8.5 ▲ The Climb Up Cost Mountain



VISUAL 8.6

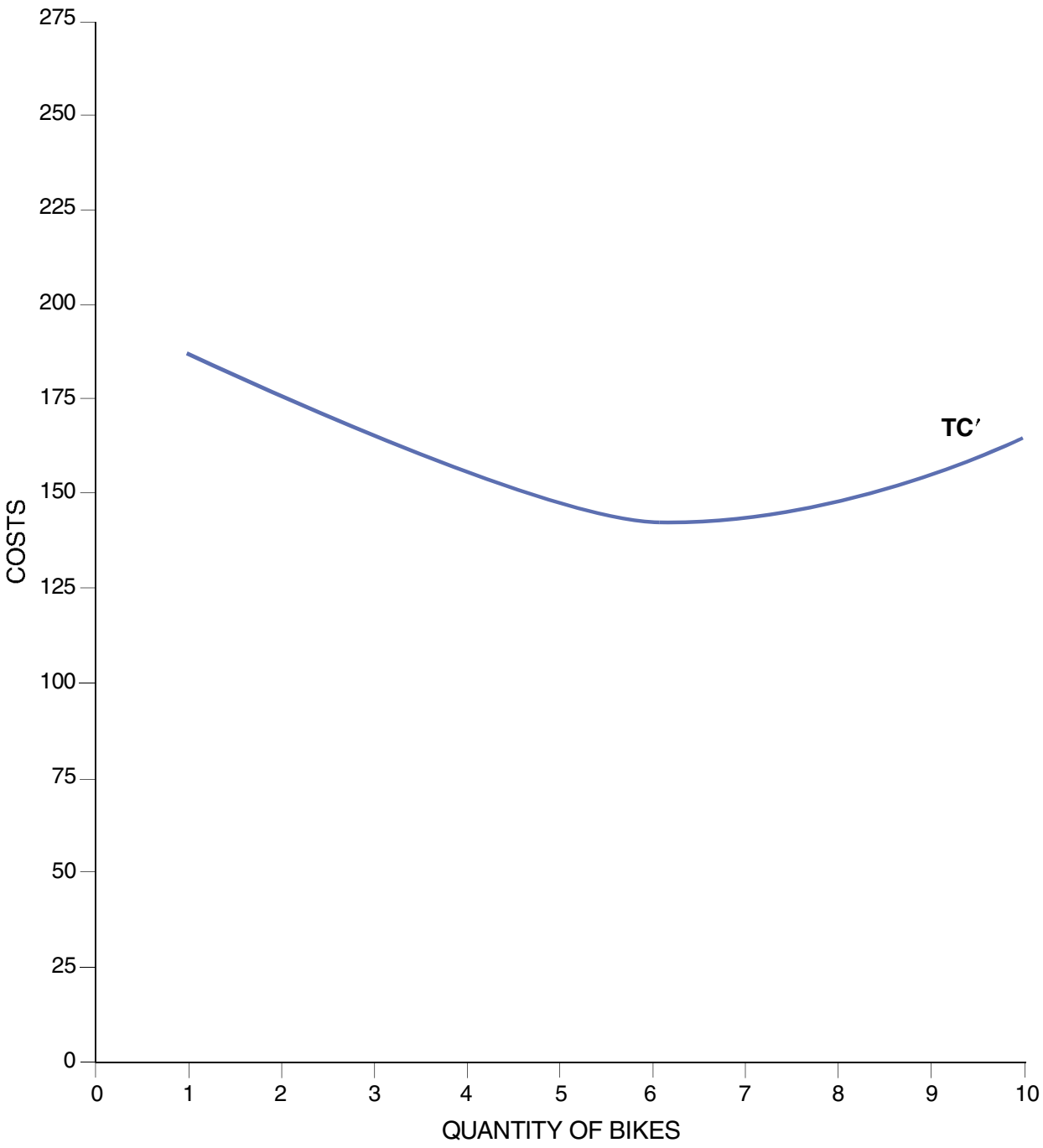

Total Cost
Total
Variable
Cost
Total Fixed Costs
COSTS
QUANTITY

VISUAL 8.7 ▲ Table Showing Total and Marginal Costs at Different Production Levels

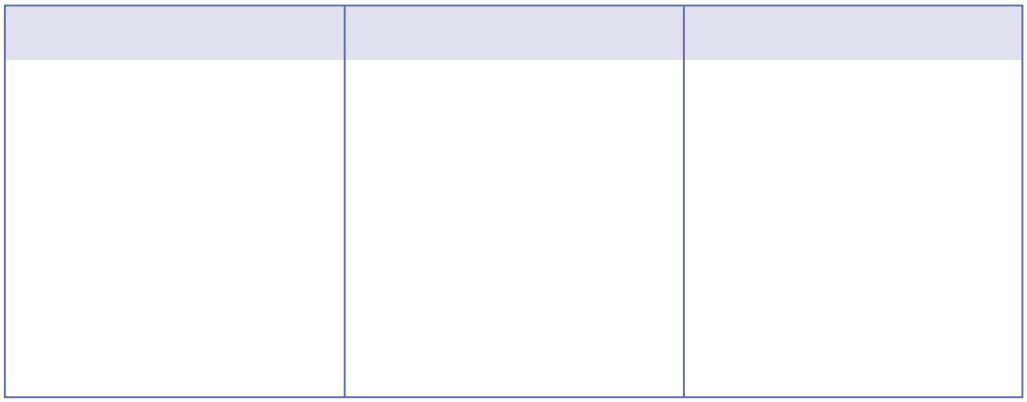


1 bikes	$291.50	$191.50
2 bikes	$468.00	$176.50
3 bikes	$632.50	$164.50
4 bikes	$788.00	$155.50
5 bikes	$937.50	$149.50
6 bikes	$1084.00	$146.50
7 bikes	$1230.50	$146.50
8 bikes	$1380.00	$149.50
9 bikes	$1535.50	$155.50
10 bikes	$1700.00	$164.50

Quantity of Bikes Made	Total Cost	Marginal Cost
0 bikes	$100.00	



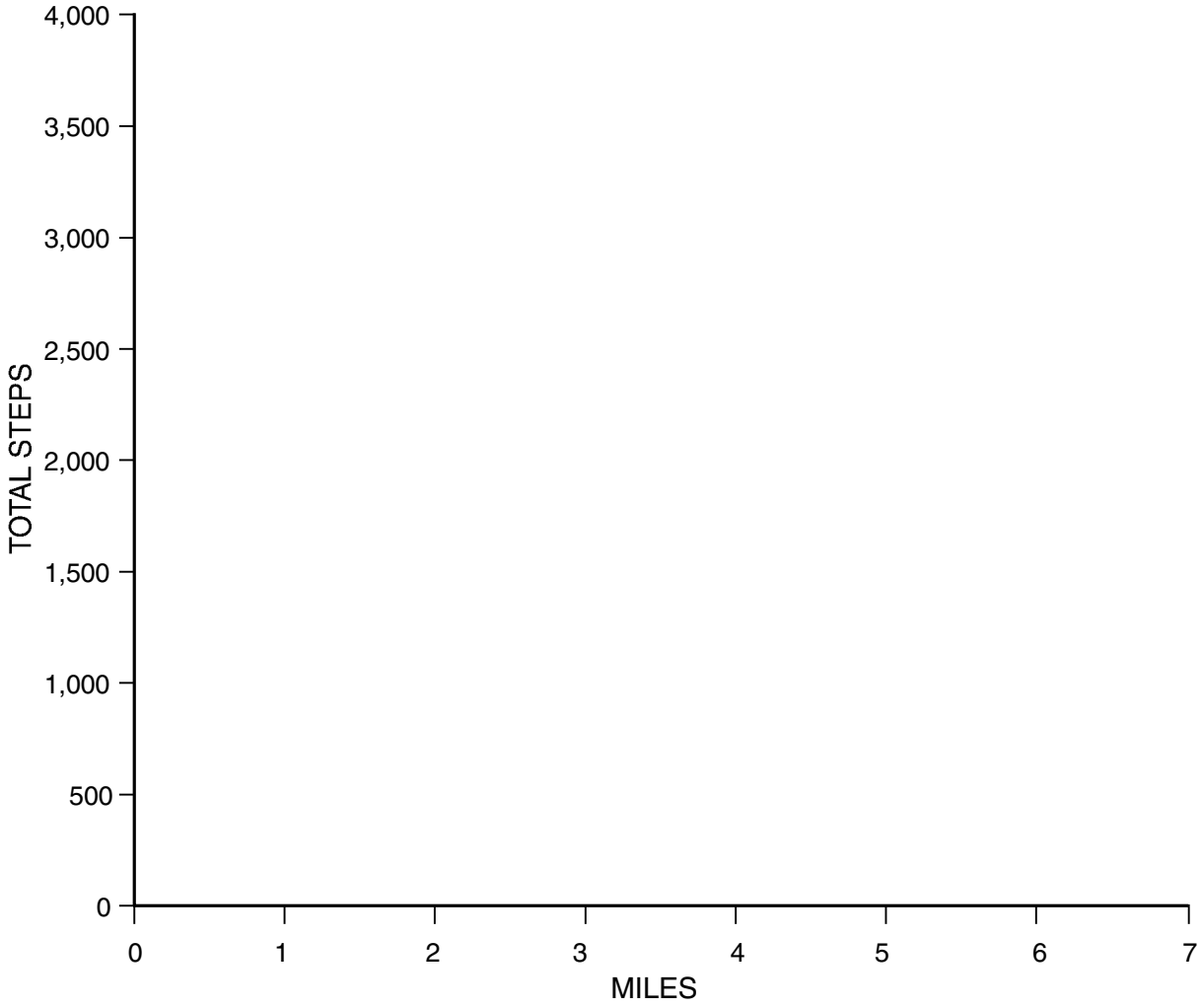
VISUAL 8.8 ▲ Marginal Cost



VISUAL 8.10 ▲ First Derivative of Total Cost Function

VISUAL 8.9 ▲ Growing the World's Food Supply in a Flowerpot

While it may be difficult to articulate, the law of diminishing marginal returns can be simply



illustrated. The classic illustration involves agricultural output in a flowerpot.

Suppose you planted corn in a flowerpot. Here the fixed factor of production is the dirt that is used in the pot (economists refer to this as the productive resource called land). Suppose you were to add fertilizer to stimulate the production of corn in your flowerpot. When you add fertilizer, you observe that you produce more corn.

The first few units of fertilizer that are added may cause accelerating corn yields, but this cannot continue forever.

Why not?

If each additional unit of fertilizer contributed to a corn yield that was greater than the previous unit, you could produce the world's supply of corn in your flowerpot. You would just have to keep adding fertilizer. This, of course, is impossible because the soil will become somewhat sour after a certain amount of fertilizer has been applied. The acidity increases as more and more fertilizer is applied. Thus, the marginal return from each additional input starts to

diminish.

The law of diminishing returns when applied to costs simply means that **marginal costs must eventually rise.**

VISUAL 8.11 ▲ Completed Table for Activity 8.4

Quantity Q	Total Cost in Dollars	Marginal Cost in Dollars
0	625	—
1	813	188
2	989	176
3	1159	170
4	1329	170
5	1505	176